3D Visualization of Paradoxes in Special Relativity – Literature Review

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ABSTRACT

The purpose of this document is to review and conclude which papers and books have been useful and beneficial to the 3rd year project of the authors, 3D Visualization of Paradoxes in Special Relativity. This document will provide a brief introduction to the project and discuss what literature was used to provide a greater understanding of the question and how it can be addressed.

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Introduction

Special Relativity

One of the greatest revolutions of our physical paradigm occurred in the early 20th century when Einstein published his paper, 'On the Electrodynamics of Moving Bodies', in 1905 [1]. By the 1900s, physicists had had experience with Newtonian mechanics for over two centuries and thought that they had almost confirmed everything there was to know about the physical world. However, there were a few things that hadn't quite been solved; one of these was the inconsistency of Newtonian mechanics with Maxwell's equations of Electromagnetism. For 40 or so years since Maxwell published his equations in 1861 [2], physicists around the world were perplexed by the constant *c* in Maxwell's equations. This particular constant just so happened to be the speed of light. The problem that confused everyone was that *c* was given without reference to any inertial observer. This didn't make much sense when Newtonian mechanics seemingly applied to everything else in the real world with reference to inertial observers. For example, if there is a person on a train passing a station throwing a tennis ball, and someone on the platform is throwing another tennis ball, assuming they both throw their respective tennis balls with the same amount of force, the tennis ball being thrown on the train will be travelling faster than the one on the platform, because its speed is the speed at which it is thrown *plus* the speed of the train. According to Maxwell's equations, this is not the case when it comes to light. If, instead of the speed of the tennis balls, it was the speed of a beam from a torch that was being measured, the speed of the beam would be exactly the same for both the train passenger and the person on the platform. This even applies if the train was travelling at 99.9% the speed of light, the light on the train would be no faster than the light on the platform in each observer's reference frame. This leads to Einstein's First Postulate of Relativity, the constancy of the speed of light [3].

Einstein assumed that Newtonian mechanics must only be a good approximation of the real world and therefore started to think up a theory of his own. Taking into account that *c* was constant in all reference frames, Einstein began to think through the repercussions of this bold postulate and the results lead to all sorts of beautifully intricate and very counter-intuitive consequences.

Time Dilation

One of these consequences is known as Time Dilation. This strange phenomenon is inherently down to an interval of time measured on a **single** clock being compared to the time difference between that clock and a **different** clock in a relative reference frame. The scale of the time interval as measured by the single clock is stretched; hence the name of the phenomenon itself [4]. The mysteriousness of the phenomenon soon dissipates once it is recognized that it is merely caused by a comparison between sequential readings on a **single** clock and measurements made by two **different** clocks.

Length Contraction

The other most common feature of Special Relativity is known as Length Contraction. In this phenomenon, the length of objects travelling at non-zero velocities relative to the observer is decreased. Length Contraction only affects the length parallel to the direction of which the object is travelling in [5]. Length Contraction can also be derived from the effects of Time Dilation [6], which has been confirmed multiple times in experimental set-ups.

Paradoxes

With these two relativistic effects in mind, a lot of thought processes become paradoxical; this is usually down to the superficial application of the contraction formula. For example, the 'ladder' paradox occurs due to the misinterpreted use of simultaneity [7]. Similarly, the Ehrenfest paradox is resolved by the concept of rigid bodies being incompatible with Special Relativity [8].

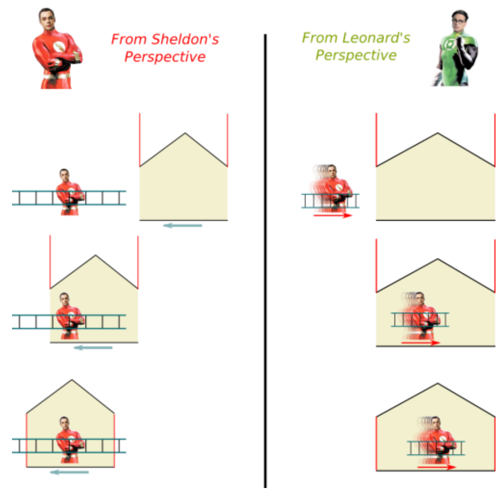

Figure 1 – The Ladder Paradox [9]

Project Outline

Project Overview

The main aim of this project is to find a way of utilizing 3D imagery so as to describe paradoxes in Special Relativity in simple terms, so that the public could look at the visualization and understand what is going on. The tasks at hand are to derive the solution of a chosen paradox and to design a 3D visualization of the derived solution.

Visualization of Paradoxes

The best way to visually express the solution to a paradox is by using a Minkowski space-time diagram. Illustrating the paradox using 2D and 3D Minkowski diagrams often allows for easier, more effective studying and understanding of the situation. Whilst Minkowski diagrams are helpful to people who are versed in Special Relativity, not everyone can understand them; therefore, another visualization method is needed to make the solution to a paradox understandable to the public. This will most likely be in the form of a simple illustration of the physical problem itself.

The Submarine Paradox (Supplee's Paradox/Bullet in Water Paradox)

In order to allow for more focus on the 3D visualization side of the project, it was decided that only one paradox be chosen. This was the Submarine Paradox (also known as Supplee's Paradox or Bullet in Water Paradox); it is different in many ways to most paradoxes in Special Relativity, this is the reason why it was chosen. The paradox is based around a scarcely covered area of Special Relativity known as Relativistic Buoyancy. The paradox arises when considering the buoyant force exerted on a relativistic submarine immersed in water subject to an ambient gravitational field. If the submarine with neutral buoyancy passes through the water at a relativistic velocity, its density increases in the water's frame of reference causing it to sink; however, the water's increased density in the submarine's frame of reference causes it to float [10]. The submarine cannot both sink and float at the same time, this is the Submarine Paradox.

Discussion of Literature

The only logical place to start the literature search was at Einstein's paper on Special Relativity itself from 1905 [11]. It proved to be a very insightful paper showing the shear brilliance of Einstein at his very best in the middle of his *Annus Mirabilis* (Miracle Year). The very premise of Special Relativity is laid out in this paper along with various equations describing the transformations in detail. However, as it is the first paper on the subject of Special Relativity, there are only a small number of references in it that lead to more useful papers. For further reading on the basics of Special Relativity, books were the way to go. Thus, a number of books were borrowed from the Physical Sciences library on the University of Aberystwyth's main campus.

The Basics of Special Relativity

AP French's "Special Relativity" (*CRC Press* 1968) was a very good benchmark to start reading. It was filled with explanations and diagrams helping the reader to understand exactly what French was trying to convey. It also had concise and useful definitions of the consequences of Special Relativity, such as Time Dilation and Length Contraction. JH Smith's "Introduction to Special Relativity" (*Dover Publications Inc.* 1996) was also crucial to understanding the basics of Special Relativity and helping the reader to comprehend the notation of Lorentz transformations. Further to the explanations from Smith, W Rindler's "Special Relativity" (*Interscience Publishers Inc.* 1960) had a very thorough mathematical approach to the workings of Special Relativity. It also had exceptionally in-depth explanations for all the phenomena of Special Relativity, albeit without many diagrams and illustrations. The literature search also lead to LD Landau's "The Classical Theory of Fields" (*Pergamon Press* 1975); however, this only covered the same material if not less than the aforementioned books as it covered a wider range of physical problems than just Special Relativity; therefore, the authors did not spend much time with this text.

Hinckfuss's "The Existence of Space and Time" (*Oxford University Press* 1975) has a more philosophical approach to explaining Special Relativity, starting with questions like 'What is Space?' and explaining the different essences of the word's meaning. This is a very good read for the general public if they are interested in this field, ideas could be taken from this book as to how to approach the explanation of resolved paradoxes in the future. There was very little mathematics in the book itself, but rather good illustrations along with thorough explanations.

Paradoxes in Special Relativity

The emphasis of the literature search was then changed to paradoxes in Special Relativity. The results of this search were many; there were dozens of different paradoxes to be studied, so the right place to start was with YQ Gu's paper "Some Paradoxes in Special Relativity" (2011) [12]. In Gu's paper relativity is treated as nothing but geometry and a number of different paradoxes are resolved, namely the Ladder paradox, Ehrenfest's rotational disc paradox, the Twins paradox and Bell's spaceship paradox. The general conclusion of the paper is that all the paradoxes are caused by a misinterpretation of the relativistic concepts themselves; in other words, the paradoxes only arise because of our unwillingness to let go of our wrongly founded ideal of global simultaneity.

While on a trip to the library, one book stood out from the others in terms of its title, and that was Terletskii's "Paradoxes in Special Relativity" (*Springer* 1968). This book proved to be very useful for learning about the paradoxes themselves but didn't really shed any light on anything that wasn't already known from reading the other books in this literature review, and therefore was a little disappointing. Had this book been found first then it might have seemed to be a little more useful. However, it did give an equation for the hyperbolae used in Rindler charts, by calling them 'Calibration Hyperbolae'.

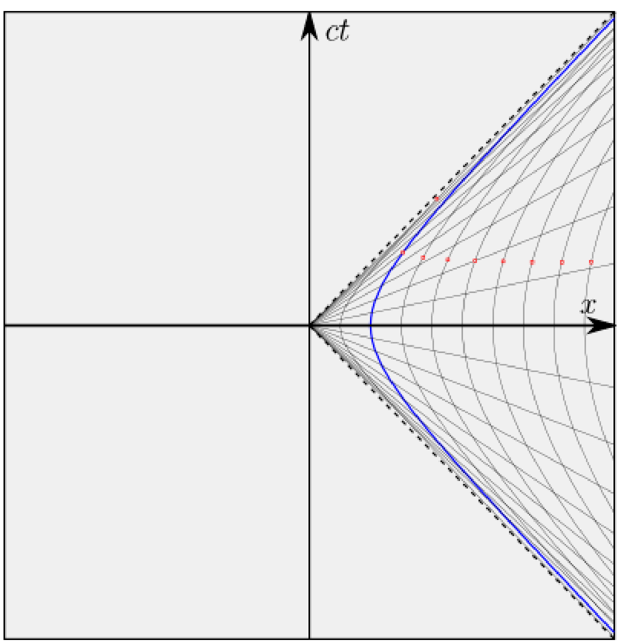

Figure 2 - Rindler Chart [13]

At first it was decided that looking into a paradox that we had already come across would prove beneficial, so that we could understand the processes in resolving it mathematically. The paradox that was chosen for this purpose was the Ladder paradox; having come across the Ladder paradox in the form of the train in the tunnel, it was very easy to comprehend this paradox and visualize it mentally. Focusing on this paradox lead to W Rindler's "Length Contraction Paradox" (1961). Rindler had resolved the Length Contraction paradox in question by creating what is now known as a Rindler chart. Rindler charts are based on Minkowski diagrams; moreover they are Minkowski diagrams for objects moving with a hyperbolic motion, using a co-ordinate system called Rindler co-ordinates, which represents a part of flat space-time.

This different approach to visualizing the paradox was very intriguing, and leads the literature search to a number of different books by Rindler.

The first book of Rindler's that was found was "Essential Relativity" (*Van Nostrand Reinhold company* 1969). This book is a very detailed account of everything to do with Relativity (Special and General). However, there was very little mention of the Rindler charts that had been seen in Rindler's 1961 paper. It became very apparent that Rindler was a very well-known scientist in the field of Relativity and that most of his books would be incredibly helpful. Rindler's "Relativity" (*Oxford University Press* 2006) is technically the 2nd edition of the aforementioned "Essential Relativity"; however it proved to be much more useful. Providing numerous pages on Rindler charts, with equations for the hyperbolae and various diagrams showing Rindler charts and their characteristics.

The Submarine Paradox

As stated in the Project Outline, one paradox had to be chosen in order to allow for more focus on the 3D Visualization of the paradox's solution. JM Supplee's "Relativistic Buoyancy" (1989) paper really stood out from the rest of the paradoxes because it was more than just a Length Contraction or Time Dilation problem; there was gravity and a buoyant force to take into account. The number of relevant papers on this subject is incredibly low in comparison to the previous success that had been seen with the other paradoxes. Supplee had stated the paradox and resolved in one short paper, and therefore it must have been assumed that no further work was needed on the matter. Supplee's 1989 paper was very self-contained; everything that was needed to understand the paradox was in the paper, along with all the mathematics used to resolve the paradox. There was however a section for discussion in which Supplee states that because gravity is involved, General Relativity should be used instead of just Special Relativity in order to fully show that the paradox is resolved.

It wasn't for another 14 years until GEA Matsas published his paper "Relativistic Archimedes law for fast moving bodies and the General-relativistic resolution of the Submarine paradox" (2003) [14]. The mathematics in this paper was very hard to follow given that the authors of this review had never studied General Relativity before. However, there is one part of the paper that is incredibly useful and that was a Rindler chart of the paradox that had been used to model the solution. Matsas came to the same conclusion as Supplee did in his paper; the submarine would sink and hit the bottom of the body of water in question and also that the shape of the body of water would change so that the bottom of the body of water comes up to meet the submarine. Therefore, allowing for the differences in density due to different reference frames while keeping the actual outcome the same.

Visualization of Paradoxes

Having found all the papers needed in order to solve the Submarine paradox, the literature search turned towards researching how the solution was going to be portrayed. This was primarily going to be done by using Minkowski diagrams and Rindler charts. Finding literature on Minkowski diagrams is exceedingly easy, being an essential part of Special Relativity naturally it is in almost every piece of literature on Special Relativity and therefore the books and papers mentioned previously in this review were suffice in explaining the concepts. Literature on Rindler charts on the other hand, is remarkably scarce. Even in Rindler's books themselves, the idea of Rindler charts wasn't given the pride of place that perhaps would have been expected. Furthermore, the majority of references to Rindler charts was in context to 2D Rindler charts. 3D Rindler charts are incredibly rare. The only reference to 3D Rindler charts that was found throughout the entire Literature search was in GEA Matsas's 2003 paper.

Additionally, there will be an illustration of the paradox created in order to help the general public understand what is actually happening in the paradox itself. The programming language in which this is to be done is yet to be determined, but there are numerous tutorials online that explain everything there is to know about 3D rendering. Once the programming language has been determined, then a few coding 'bibles' shall be selected such as K Sierra's "Head First Java" (*O'Reilly Media* 2005) or J Zelle's "Python Programming: An Introduction to Computer Science" (*Franklin, Beedle & Associates Inc.* 2010). G Gamow's "The New World of Mr. Tompkins" (*Cambridge University Press* 1999) provided a great deal of ideas on how to illustrate the strange world of Special Relativity, the protagonist of the book travels to a world where the 'speed limit of nature' is in the region of 20 mph and therefore there are many different visual effects that the protagonist can see; for instance when he observes someone on a bike, or when he himself rides a bike.

Figure 3 – Length Contraction in "The New World of Mr. Tompkins" [15]

Conclusion

In conclusion to this literature review, many valuable sources of information were found in the literature search. The basics of Special Relativity are very widely and readily available in countless books, papers, and articles and are now fully understood by the authors. Papers on paradoxes in Special Relativity are numerous but only a few get a good deal of coverage, namely the Ladder paradox, Ehrenfest's paradox and the Twins paradox. The paradox chosen for this project is the Submarine paradox (also known as Supplee's paradox or Bullet in Water paradox). There are only a couple of papers on this paradox, but they are incredibly concise and extremely useful. Literature on Rindler charts is scarce but Minkowski diagrams (that Rindler charts are based on) are well documented.

Acknowledgements

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